International Workshop on Coupled Methods in Numerical Dynamics
IUC, Dubrovnik, Croatia, September 19th-21st, 2007

Linear Algebra for Solving Large Sparse Systems Occurring in Computational Fluid Dynamic

Jean–Marie Marchal*

C.A.E. Consulting, Belgium

Abstract. We present the three families of linear algebra solvers currently in use in the context of CFD, namely (algebraic) multigrid, direct solvers and Krylov techniques. They are presented in the unified framework of projective techniques and "divide and conquer" strategies. Their merits and drawbacks for the various kinds of matrices currently found in implicit simulation problems are discussed.

Key words: *Linear algebra, Multigrid, AMG, Direct solvers, Krylov solvers, Heuristics.*

1. Introduction

Within the field of Computational Fluid Dynamics and more generally numerical simulation, linear algebra is certainly not the most popular discipline, at least from the number of publications and general interest point of view. Quite frequently, the emphasis is on the development of additional physical models or on discretization techniques. With the notable exception of fast (and usually compressible) flows (where changes take "time" to propagate to the complete domain, at the time scale of the simulation), linear algebra is however absolutely critical in terms of CPU and memory performances for most applications. This is due to steady-state or quasi-steady-state nature of many engineering processes, where the required implicitness automatically implies solving a large and sparse system of equations. Most of the time, those simulations spend 90% of their time in linear algebra, not to mention memory requirements.

Quite generally, the problem is formulated as:

$$\begin{aligned}&\text{Find } x \in X \text{ such that}\\ &Mx = b\,,\end{aligned} \tag{1}$$

where X represents a *finite-dimensional vector space* in which the solution is to be looked for. X represents whatever the solution variables might be, for example velocity, pressure, temperature or additional variables. Unfortunately, linear systems formulated as in (1) are of little use as without additional knowledge

*Correspondence to: CAE Consulting, Louvain-la-Neuve, Belgium, `jmm@caeconsulting.com`.