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Geometric Modeling of Multibody Systems

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Abstract. A formulation for the kinematics of multibody systems is presented, that uses Lie group concepts. With line coordinates the kinematics is parameterized in terms of the screw coordinates of the joints. Thereupon, the Lagrangian motion equations are derived, and explicit expressions are given for the objects therein. It is shown how the kinematics and thus the motion equations can be expressed without the introduction of body-fixed reference frames. This admits the processing of CAD data, which refers to a single (world) frame. For constrained multibody systems, the Lagrangian motion equations are projected to the constraint manifold, which yields the equations of Woronetz. The Boltzmann-Hamel equations are recalled as an alternative formulation in terms of non-holonomic velocities.

Key words: *Multibody systems, Lagrange equations, Bolzmann-Hamel equations, Screw systems, Lie groups, Coordinate invariance, CAD.*

1. Introduction

Geometric methods in dynamics and control have become widely established and valued approaches. The actual meaning of 'geometric' varies with the context it is used. It generally refers to the differential geometry of the mathematical structures underling a dynamical system. As an example, non-linear control theory for continuous systems is solely built upon the differential-geometry of the control system [3],[19],[20],[21]. In other words, generally, one deals with control systems on manifolds. Another area that makes use of geometric concepts, is the field of geometric integration [8]. The subject here is the integration of dynamical systems on manifolds, of which systems on Lie groups are special cases [25],[10].

In mechanism theory one is concerned with the geometry of finite motions, and on velocity level with their differential geometry. As it turns out in the case of rigid multibody body systems (MBSs), the differential-geometric objects, are in correspondence with the geometry of the joints and bodies. The underlying reason is the isomorphism of the algebra of screws and the algebra of the Lie-group of rigid body motions [18],[32]. The latter is the group of isometric orientation preserving transformations of the three-dimensional Euclidean space. In essence, the parameterization of screws with line coordinates gives rise to a

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