

# Numerical Constrained Mechanics of Discrete Systems on Manifolds

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**Abstract.** Numerical forward dynamics of discrete mechanical systems with constraints is in focus of the paper. During dynamical simulation of constrained mechanical systems, a numerical violation of kinematical constraints is the basic source of integration errors. Mathematical models of constrained discrete systems are surveyed and geometric properties of constraints are discussed. The “mechanisms” of emerging of constraint violation errors and numerical integration errors along system configuration manifold are discussed. The stabilized time-integration procedure, whose stabilization step is based on projection of the integration results to the underlying constraint manifold via post-integration correction, is investigated. After discussing optimization of the partitioning algorithm, the geometric and stabilization issues of the method are addressed and it is shown that the projective algorithm can be applied for numerical stabilization of holonomic and non-holonomic constraints in Pfaffian and general form. As a continuation of the previous work, a further elaboration of the projective stabilization method applied on non-holonomic discrete mechanical systems is reported and numerical example of dynamical simulation of satellite solar array deployment is provided.

**Key words:** *Dynamic simulation of discrete mechanical systems, Multibody systems, Constraint violation stabilization, Numerical dynamics on manifolds.*

## 1. Introduction

During dynamical simulation of constrained multibody systems, a violation of system kinematical constraints is the basic source of time-integration errors and frequent difficulty that analyst have to cope with. If the governing equations are not turned into so called minimal form, but dynamic simulation is based on mathematical models expressed via redundant coordinates, a constraint violation stabilization method has to be applied during integration procedure. Baumgarte stabilization method that minimizes violations can be applied for this purpose,

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